N- AND S-MODES OF SELF-SIMILAR COMPRESSION OF A FINITE MASS OF PLASMA AND PROPERTIES OF THE MODES WITH PEAKING

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The use of laser radiation to heat a plasma to superhigh temperatures was suggested in [1]. Research in the field of laser thermonuclear synthesis (LTS) was particularly intensified after a proposed new approach to the solution of this problem [2], the essential feature of which is the requirement that the flux q(t) of laser radiation incident on the surface of the DT target vary in a peaking mode, i.e., $q(t) \to \infty$ monotonically at $t \to t_f$, t_f being finite. The purpose of the present report is to bring out a number of properties which are common to processes in which modes with peaking appear. The authors do not pretend to any complete survey of reports devoted to different aspects of the LTS problem. The survey begins with a clarification of the question of how the law of time variation of the laser radiation intensity providing for close to adiabatic compression of the central part of a DT drop is obtained in [2-5]. Then we analyze a number of processes characteristic of the development of modes with peaking in a continuous medium, the significance of which, in the author's opinion, is far wider than their use in the LTS problem. Modes with peaking are connected with the properties of strongly nonsteady processes, which are characterized by the phenomenon of metastable localization of processes of transfer of heat, the magnetic field, and other quantities in certain sections of the medium. Stable temperature and other inhomogeneities (structures) develop in the medium as a result.

In the present report attention is concentrated on a survey of the results of the investigation of the properties of the thermodynamics of modes with peaking which are due to the appearance and development of structures in the continuous nonlinear medium.

The problem of the compression of a half-space by a plane piston was analyzed in [6] and a pressure law providing for compression without shock waves was obtained: $p(r_p, t) \sim (t_f - t)^{-2\gamma/(\gamma+1)}$.

A solution exists up to the time $t = t_f$ when the characteristic curves intersect at one point (r = 0). The radius r_p of the piston measured from this point decreases in accordance with the law $r_p \sim (t_f - t)^{2/(\gamma+1)}$.

In [2] the law q(t) for the laser flux used for almost adiabatic compression of a drop was chosen from the following considerations. It was assumed that the time dependence of the velocity in the adiabatically compressed region behind a weak shock wave front is obtained from $v \sim \sqrt{[2/(\gamma + 1)](p/\rho_0)} \sim \sqrt{p}$ (γ is the adiabatic index and ρ_0 is the density ahead of the shock wave). The time dependence of the flux of laser radiation is obtained from dimensionality relations for the case of a plane geometry: $q \sim E \sim pv \sim p^{3/2}$. The pressure dependence is taken from the problem of the adiabatic compression of a half-space by a plane piston which was solved in [6]. Then for the flux one obtains $q \sim (t_f - t)^{-3\gamma/(\gamma+1)}$. With $\gamma = 5/3$ we have $3\gamma/(\gamma + 1) = 1.875$. This time dependence of the flux is presented in [2]. It is noted that the law $q \sim (t_f - t)^{-2}$ is usually used in numerical experiments on the spherical compression of a drop.

The assumption of the homogeneous compression of a finite mass of plasma was made in [5], i.e., it was assumed that the law $r = r_0 h(t)$ of the time variation of the radius of any element of mass is the same, where r_0 is the position of this radius at the initial time. Such an assumption of the separation of the Lagrangian (mass) and time variables made it possible to obtain from the momentum equation an equation determining h(t),

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$$d^2h/dt_*^2 = -h^{2-3\gamma},$$

where $t_* = t/t_f$. In this case it was assumed that $p \sim \rho^\gamma \sim (M_0/r^3)^\gamma \sim h^{-3\gamma}$, where $M_0 =$ const, i.e., the mass of the ball being compressed was taken as constant. The general solution of the equation for h(t) is as follows:

$$\int \frac{dh}{\sqrt{C_1 - 2\int h^{2-3\gamma} dh}} = -t_* + C_2.$$

The choice of the integration constants C_1 and C_2 determines the two types of solution of the problem. Taking $C_1 = 0$ and $C_2 = 1$, which corresponds to a nonzero velocity at the initial time (t = 0), one can obtain

$$h_1(t) = \left(\frac{3\gamma - 1}{\sqrt{6(\gamma-1)}}\right)^{\frac{2}{3\gamma-1}} (1 - t_*)^{\frac{2}{3\gamma-1}}.$$

The other case, when the velocity of the piston equals zero and h(0) = 1 at the initial time, is analyzed in [5]. Taking $C_1 = 2/3(1 - \gamma)$ and $C_2 = 0$ for this, the author finds the following solution for the case of $\gamma = 5/3$: $h_2(t) = (1 - t_*^2)^{1/2}$. We note that $h_1 = \sqrt{2}(1 - t_*)^{1/2}$ when $\gamma = 5/3$. As $t \to t_f$ we have $h_2 = (1 + t_*)^{1/2}(1 - t_*)^{1/2} \to \sqrt{2}(1 - t_*)^{1/2}$, i.e., $h_2 \to h_1$. In [5], similarly to [2], it was assumed that $v \sim \sqrt{p}$, and then the following law for the total laser flux was obtained in [5] with allowance for sphericity:

$$q \sim r^2 pv \sim r^2 p^{3/2} \sim h_2^{-11/2} \sim (t_f^2 - t^2)^{-11/4}.$$

Finally, in [3, 4], also on the assumption of constancy of the mass being compressed and using dimensionality relations analogous to the preceding ones (solutions of the type of h_1 were used), the laws

$$r \sim (1 - t_*)^{2/(3\gamma-1)}, \; p \sim \rho^\gamma \sim r^{-3\gamma} \sim (1 - t_*)^{-6\gamma/(3\gamma-1)}$$

and the time dependence of the total laser radiation flux

$$q \sim r^2 pv \sim r^2 p\,dr/dt \sim (1 - t_*)^{(7-9\gamma)/(3\gamma-1)}$$

were obtained with $\gamma = 5/3$ and $q \sim (t_f - t)^{-2}$. In these reports, in contrast to [2, 5], it was assumed that the law of time variation of the velocity must be taken from v = dr/dt.

The problem of adiabatic compression of a plasma (compression without the formation of shock waves in it) has been strictly solved in an analysis of the problem of compression of a finite mass of plasma by a piston. Such a problem admits of a self-similar formulation for one-dimensional nonsteady equations of gasdynamics in the cases of plane, axial, and central symmetry. An analytical solution for the case of adiabatic compression is constructed below. This solution is based on the use of the separation of the spatial (mass) and time variables in the initial problem for the equations of gasdynamics. With such an approach no waves of finite amplitude propagating through the mass, including shock waves, can appear. It was shown in [7] that for adiabatic compression of a plasma the laws of time variation of the radius to the converging piston and of the pressure at it will be the following: $r_p \sim t^n$, $p_p \sim t^{-n(N-1)-2}$. In a self-similar formulation for problems of compression the time varies from $t = -\infty$ to t = 0 (the time t = 0 corresponds to the time of focusing of all the mass at the center) and the index n in this case of adiabatic compression equals $n = 2/[2 + (N + 1)(\gamma - 1)]$, where γ is the adiabatic index ($\gamma = c_p/c_V$) and N = 0, 1, and 2 for the cases of plane, cylindrical, and spherical symmetry, respectively. The law of time variation of the total heat flux (W) or total laser flux $q = q_0 t^{2n-3}$ follows directly from dimensionality considerations in this case. In the case of a plane piston (N = 0) the law for the pressure at it, obtained in [6] by the method of characteristic curves of the gasdynamic problem, coincides with that presented here. In the case of a spherical piston we obtain the laws indicated in [2], which are found in [3, 4] using dimensionality relations.

For N = 2 and $\gamma = 5/3$ the cited laws give $r \sim t^{1/2}$, $p \sim t^{-5/2}$, and $q \sim t^{-2}$. In order to realize self-similar modes starting with $t = -\infty$, in a computer calculation of the corresponding problem in partial derivatives, for example, the time of focusing $t \to t - t_f$ was shifted in [7-13]. In this case the self-similar mode begins at $t = -\infty$ and ends at the time of focusing $t = t_f$ ($-\infty < t < t_f$). The mode actually realized began at t = 0 ($0 \le t < t_f$). With such an approach the laws of adiabatic compression presented above have the form

$$r \sim (t_f - t)^{1/2},\ p \sim (t_f - t)^{-5/2},\ q \sim (t_f - t)^{-2}.$$

Here instead of p and q being equal to zero at $t = -\infty$, finite values of $p \sim t_f^{-5/2}$ and $q \sim t_f^{-2}$ are "turned on" with a jerk at the initial time, which gives rise to a weak shock wave cumulating at the center and altering the initial state (the entropy, in particular) of the plasma. One can get rid of the initial shock wave if one assigns a self-similar mass distribution at the initial time, as shown in [5, 7], for example. The larger t_f, the smaller the initial "jerk" and the closer the problem to being self-similar and the solution to the analytical solution presented below for the adiabatic case. In reality, the energy of the laser pulse is finite. Consequently, the radiation flux is cut off at $t = t_1 < t_f$. One can take different t_1 and t_f for one and the same energy of the laser pulse [for one and the same law $q \sim (t_f - t)^{-2}$]. Numerical experiments show that an increase in t_f, while it does weaken the initial shock wave, simultaneously decreases the limiting pressure and density reached in the calculation.

The thermal wave which compresses the plasma in numerical experiments of "laser squeezing" of a drop is replaced by a piston in the self-similar formulation. The connection of the problem of a piston with the problem of the "laser squeezing" of a plasma drop consists in making the assumption that the pressure in the thermal wave front must vary by the same optimal law as that at a real piston, in order to provide for the strong (almost adiabatic) compression of the part of the mass ahead of the front of the subsonic thermal wave. The results of numerical calculations confirm the possibility of such "supercompression."

The possibility of constructing a solution in separable variables also in the case when dissipative processes are taken into account in a finite mass of plasma compressed by a piston is demonstrated below, and volumetric sources and sinks of energy (due to volumetric emission) characteristic of a fully ionized plasma are also modeled. In this case the compression is also accomplished by sound, shock waves do not develop in the medium, and all the effective fronts (regions of large gradients) of thermal or other waves do not move with respect to mass. The latter assertaion, although it seems paradoxical, follows directly from the possibility of separating the variables in the complex nonlinear system of equations describing the motion of the fully ionized plasma. Proof of the possibility of "stopping" of the thermal front (with a null initial background) in the problem of heat propagation in a medium with a coefficient of thermal conductivity which increases with temperature was first obtained in [14].

The physical meaning of solutions with a standing effective thermal front can be explained as follows. The effective depth of penetration (Δx_T) of heat through the mass in a time t is determined by the equation $\Delta x_T \sim \sqrt{\varkappa_M t}$, where $\varkappa_M$ is the mass coefficient of thermal conductivity, which depends on the temperature, density, and radius. (For example, in the case of a fully ionized plasma and spherical symmetry the heat flux described in mass Lagrangian coordinates has the form $W = -\varkappa_0 T^{5/2} r^4 \rho dT/\partial x$. In this case $\varkappa_M = \varkappa_0 T^{5/2} \rho r^4$.)

In the process of a self-similar mode of compression all the functions vary with time in such a way that the time dependence of $\varkappa_M$ always has the form $\varkappa_M \sim 1/t$. Hence it follows that in these modes the effective depth of penetration of heat through the mass is constant and does not depend on time: $\Delta x_T \sim \sqrt{(1/t)t} = \text{const}$.

For problems of the self-similar compression of a finite mass of plasma the time varies in the range of $-\infty < t < 0$. Thus, $\varkappa_M$ grows (like the temperature and density) as t approaches the moment of focusing ($t \to 0$). It is shown in [7-13] that such compressions (without shock waves) of a finite mass of fully ionized plasma by a piston are possible with laws analogous to those indicated above for adiabatic compression but now with an index $n = 4/(4 + N)$. For the case of $N = 2$ we have $n = 2/3$, and the law of variation of the total laser flux will be $q \sim t^{-5/3}$ $(-\infty < t < 0)$ or $q \sim (t_f - t)^{-5/3}$ $(0 \le t < t_f)$. The law of growth $q = q(t)$ in the case of the mode of compression by sound (without shock, thermal, or other waves of finite amplitude) of a finite mass of fully ionized plasma (S-mode) is somewhat slower (as $t \to t_f$) than the law for adiabatic compression (N-mode). The N-mode is usually used in numerical experiments on the squeezing of a drop by laser radiation. Because of this the thermal wave moves through the mass in the N-mode. As noted in [2], with the motion of the thermal wave through the mass the symmetry and stability of the plasma compression by its front are considerably improved in comparison with the compression accomplished by a piston.

From this it is seen that the range of variation of the exponent g in the law $q = q_0 t^g$ in the transition from the N-mode to the limiting S-mode is not wide: $-2 \le g \le -5/3$ (in the case of $N = 2$ and $\gamma = 5/3$).

Two analytical solutions are given in the present report: one for the problem of the adiabatic compression of a finite mass of plasma by a piston and the other for compression of a fully ionized plasma by a piston moving with a constant velocity.

They make it possible to analyze the characteristic features of the modes and their dependence on a number of parameters of the problem. The analytical solutions are convenient for demonstrating the principle of the transition from problems of the dispersion of a finite mass of plasma to problems of its compression. The principle itself was first formulated in [8] and has been widely used for the study of different modes of compression in [7-13].

The analysis of self-similar solutions with separable variables in the problems of the compression of a fully ionized plasma makes it possible to clarify another interesting property of these solutions. In the present report (see also [7, 9-13]) examples are cited and the conditions are clarified for the case when the profiles of the quantities in the plasma being compressed are spatially nonmonotonic in the process of the monotonic compression of the plasma by a piston with monotonic thermal conditions at it. For example, several temperature maxima exist. The appearance of such solutions (we will call them solutions with structures) is due to a superheating instability in a number of cases [15, 16]. The structures represent the developed nonlinear stage of such instabilities. The conditions for the appearance of such structures in self-similar modes of compression of a finite mass of plasma due to viscous dissipation, owing to the temperature dependence of the coefficient of ionic viscosity, are analyzed in [7, 11]. Their formation can also be due to the presence in the plasma of volumetric sources and sinks of heat which depend on the temperature and density (due to volumetric emission from the plasma, for example). Viscous structures were discovered in numerical experiments in [17].

Structures due to volumetric emission were studied in the steady-state Z-pinch problem in [18]. In a number of cases the appearance of structures in the plasma can adversely affect the modes of its compression by laser radiation. Their appearance must lead to an increase in entropy in the medium and hinder its compression. To some extent this process is analogous in its consequences to the process of heating of the compressed central parts of the plasma by fast electrons generated in the region of absorption of the laser radiation in the corona. The latter process has been studied intensively by a number of investigators and, as indicated in [2, 19], can have an essential effect on the compression and burning of the drop in certain cases. The explanation of the role of superheated structures in the process of compression and burning of a drop requires the conducting of the appropriate numerical experiments.

On the other hand, the formation of structures can signify the possibility of heating individual sections of the plasma to temperatures markedly exceeding the average, and of initiating a thermonuclear reaction under conditions when the average temperatures are insufficient for its occurrence.

Structures also develop in the presence of a magnetic field in the plasma. They can be produced by a superheating instability on Joule heat: These are the so-called current layers, which have been studied in numerical and physical experiments [20-33]. But there are also a number of other specific magnetic structures which are responsible for the breakup of the plasma into regions with direct and reverse currents and the formation of layers of magnetization in the plasma. Singular structures develop in certain cases in a plasma in regions where there are zero values of the magnetic field strength. In [20-22, 24] in the self-similar problem of the dispersion of a finite mass of plasma examples of current layers were constructed and their properties were studied. Examples of the magnetic structures indicated above were constructed in [9-13] and a number of their properties were determined in the self-similar problem of the compression of a finite mass of plasma by a piston under the conditions when there is a magnetic field in the plasma. It was also possible to construct self-similar modes with separable variables for the compression stage in classical problems of Z- and Θ-pinches. Analytical solutions have been obtained in a number of cases. The construction of such modes means that one can also indicate the laws of variation of the hydrodynamic, thermal, and magnetic quantities at the piston such that the compression of a fully ionized plasma will take place without the propagation of any waves of finite amplitude (shock, thermal, magnetic) through the mass, i.e., in the S-mode, for the case when there is a magnetic field in the plasma. Such a mode of compression was first proposed in [8], where it was shown that compression of the plasma in a Θ-pinch by sound (without shock or other waves) is accomplished in the case of a fully ionized plasma when the magnetic field varies according to the law $H \sim (1/t)(-\infty < t < 0)$ or $H \sim 1/(t_f - t)$ for $0 \leq t < t_f$.

In [9] it was shown that in a Z-pinch it is required that the profile of the rise of the total current with time follow the law $I = I_0 t^{-1/5}$ $(-\infty < t < 0)$ or $I = I_0 (t_f - t)^{-1/5}$ $(0 \leq t < t_f)$. In this case the compression of the column of fully ionized plasma also takes place without the propagation of shock, thermal, or magnetic waves of finite amplitude through it, although in the plasma one takes into account the exchange between the ion and electron temperature, the electronic and ionic heat conduction, the finite conductivity, the ionic viscosity, and volumetric sinks modeling the volumetric emission.

The adiabatic mode of plasma compression in Z- and Θ-pinches was analyzed in [34], where the modes of the rise in the current $I(t)$ and the external field $H(t)$ were found numerically in the form of solutions of the h_2 type. The presence of a magnetic field in the plasma was modeled by the value $\gamma = 2$ in the purely gasdynamic problem. The solutions found approach $h_1(t)$ as $t \to t_f$, as in the case of the compression of a ball. The limiting case of adiabatic compression (solutions of the h_1 type) in pinches was examined analytically in [9].

Thus, the self-similar modes of compression of a finite mass of plasma by a piston (or modes with separable variables) constructed in [7-13] allow one to obtain and study from a unified point of view the modes of plasma compression by small sonic disturbances both in the case of its adiabatic compression (N-mode) and with allowance for the complicated sum of the dissipative processes in a fully ionized plasma (S-mode).

On the other hand, the possibility of using the indicated class of self-similar solutions to analyze the developed nonlinear stage of superheated instabilities and some other instabilities in a plasma proves to be no less interesting. The distinctive physics of a plasma containing structures arises at this stage.

1. Statement of the Self-similar Problem

We will seek in separable variables (the time variable and the Lagrangian mass coordinate) the solution of the problem of the compression of a finite mass of plasma $[2\pi N + (1/2)\cdot(2 - N)(1 - N)]M_0$ by a piston, described by a one-dimensional nonsteady system of gasdynamic equations with allowance for the electronic and ionic thermal conductivities ($\varkappa_e$, $\varkappa_i$), the first and second ionic viscosities (η, ζ), and the exchange of energy between the ion and electron components $[Q = \xi(T_i - T_e)]$ and between volumetric heat sources and volumetric emission [7]. The following boundary conditions are assigned: At the center the velocity $v = 0$ and the heat flux $W_{i,e} = 0$, at the piston the velocity $v = v_0 t^{n-1}$ and the heat flux $W_{i,e} = M_0 v_0^2 \omega_{i,e}(s_*) t^{2n-3}$ or the pressure $p = p_0 t^{-n_1}$ and $q = q_0 t^{2n-3}$, where $q = W_i + W_e$ and $n_1 = n(N - 1) - 2$. The dissipation coefficients have a power-law form: $\varkappa_e = a_1 T_e^{m_1} \rho^{k_1}$; $\varkappa_i = a_2 T_i^{m_1} \rho^{k_2}$; $\eta = a_3 T_i^{m_3} \rho^{k_3}$; $\xi = a_4 T_e^{m_4} \rho^{k_5}$; $\zeta = a_5 T_i^{m_5} \rho^{k_4}$; The plasma is assumed to be ideal: The pressure is $p = p_i + p_e = \rho R(T_i + zT_e)$ and the internal energy $\varepsilon = p/\rho(\gamma - 1)$, where R is the gas constant.

The volumetric heat sources are

$$Q_i = \frac{a_6 T_i^{m_6} \rho^{k_6}}{1 + a_8 T_i^{m_8} \rho^{k_8}}; \quad Q_e = \frac{a_7 T_i^{m_7} \rho^{k_7}}{1 + a_9 T_i^{m_9} \rho^{k_9}}.$$

The sink due to volumetric emission is

$$Q_E = a_{10} T_e^{m_{10}} \rho^{k_{10}}.$$

The self-similar variable is $s = x/M_0$ and the condition of self-similarity is $n = L_i/K_i = (L_j - g)/K_j = (L_k + 1)/(K_k + 1 - N)$, $i = 1, 2, \ldots, 5$, $j = 6, 7, 10$, $k = 8, 9$; $L_i = 2m_i - 1$, $i = 1, 2, \ldots, 10$; $K_i = 2m_i - k_i(N + 1) + N - 1$, $i = 1, 2, \ldots, 10$.

Self-similar modes with a parameter of finite mass (regular mode) were analyzed earlier in [35-37]. The introduction of negative time for the analysis of adiabatic problems of compression was proposed for the classical problem of the cumulation of a shock wave [38, 39].

2. Analytical Solution for Adiabatic Compression of a Plasma by a Piston

In the case of an adiabatic process ($p = \Sigma_0 \rho^\gamma$) the problem has an analytical solution. The exponent in the law of motion of the piston with time $[r_p = (v_0/n)t^n]$ in an adiabatic mode with separable variables (N-mode) is $n = n_* = 2/[2 + (N + 1)(\gamma - 1)]$. The time dependence of the quantities in the N-mode is as follows: For the temperature $T = r_*^2 t^{-2} R^{-1} \Theta(s)$, for the velocity $v = r_* t^{-1} \alpha(s)$, for the pressure $p = r_*^{(1-N)} t^{-2} M_0 \beta(s)$, for the density $\rho = r_*^{-(N+1)} M_0 \delta(s)$. Here $r_* = v_0 t^{n_*}$. The dimensionless quantities $\Theta(s)$, $\alpha(s)$, $\beta(s)$, and $\delta(s)$ depend on the dimensionless mass coordinate $s = x/M_0$, which is connected with the dimensionless spatial coordinate $\lambda = r/v_0 t^n$ by the condition $s = \int_0^\lambda \delta\lambda^N d\lambda$. Converting from s to λ, the analytical solution of the self-similar system of ordinary differential equations can be written in the form

$$\Theta(\lambda) = \sigma_0 \delta^{\gamma-1}(\lambda); \ \alpha = n_* \lambda; \ \beta(\lambda) = \sigma_0 \delta^\gamma(\lambda);$$

$$\delta(\lambda) = \left\{\frac{(N+1)(\gamma-1)^2 (C_1 + \lambda^2)}{\sigma_0 \gamma [(N+1)(\gamma-1)+2]^2}\right\}^{\frac{1}{\gamma-1}},$$

where $\sigma_0 = \sum_0 v_0^{-2/n_*} M_0^{\gamma-1}$. The constant C_1 is determined from the condition $\int_0^{1/n_*} \delta\lambda^N d\lambda = 1$. The requirement that $C_1 \geq 0$ results from the condition $\Theta > 0$ in the interval $0 < \lambda < 1/n_*$, which imposes a restriction on the quantity σ_0: $\sigma_0 \geq \sigma_0^*(N, \gamma)$. Therefore the N-mode occurs with an additional condition on the constant in the law of time variation of the piston velocity:

$$v_0^{2/n_*} \leqslant \frac{\sum_0 M_0^{\gamma-1}}{\sigma_0^*(N, \gamma)}.$$

3. Analytical Solution of the Problem of Rarefaction or Compression of a Heat-Conducting Gas in the Case of Uniform Piston Motion

Let the coefficient of thermal conductivity be $\varkappa = a T^m \rho^{N/(N+1)}$, and then n = 1 in the law of piston motion $r_p = (v_0/n)t^n$. The solution has the following form:

temperature

$$T(r, t) = \frac{v_0^2}{R} \Theta(r/v_0 t) \equiv \frac{v_0^2}{R} \Theta(\lambda),$$

$$\Theta(\lambda) = \left\{\left(C_1 + \frac{\beta_0^{\frac{1}{1+N}} \lambda^2}{2A}\right)\left(m + \frac{1}{N+1}\right)\right\}^{\frac{1+N}{m(1+N)+1}};$$

density

$$\rho(r, t) = \frac{M_0 \beta_0}{(v_0 t)^{nN+1} \Theta(r/v_0 t^n)};$$

pressure

$$p(r, t) = M_0 (v_0 t)^{1-N} t^{-2} \beta_0.$$

Conditions at the center:

$$v(0, t) = 0; \ \varkappa(\partial T/\partial r)(0, t) = 0;$$

at the piston:

$$v(r_p, t) = v_0 < 0; \ T(r_p, t) = (v_0^2/R)\Theta.$$

The integration constants β_0 and C_1 are determined from the conditions

$$\Theta(1) = \Theta_p; \ \int_0^1 \frac{\beta_0 \lambda^N}{\Theta(\lambda)} d\lambda = 1, \ N = 0, 1, 2,$$

in the cases of plane, cylindrical, and spherical symmetry. Compression corresponds to times of $-\infty < t < 0$ and expansion to times of $0 < t < \infty$.

For the compression mode ($-\infty < t < 0$, $A < 0$) the entropy decreases with time, the dimensional heat flux is directed outward and increases in magnitude with time, and the temperature

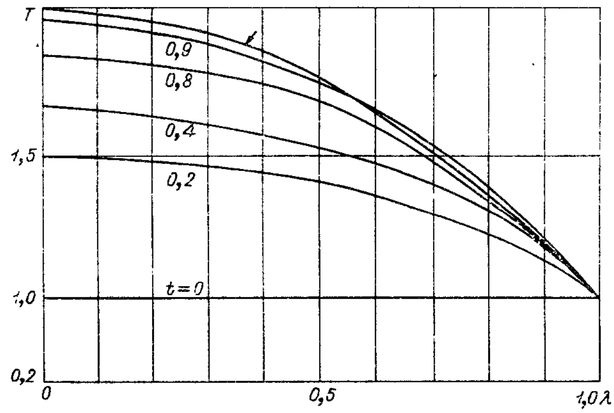

Fig. 1

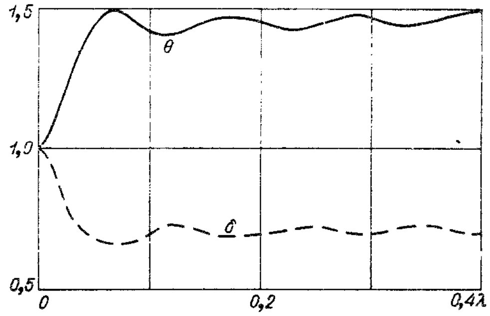

Fig. 2

profile decreases from the center to the edge. In the rarefaction mode ($0 < t < \infty$, $A > 0$) the temperature profile increases up to the piston, the entropy of the mass increases, and the dimensional heat flux is directed into the compressed mass and decreases in magnitude with time. The exact solution with the values of the parameters $m = k = 2/3$, $A = -1/2$, $N = 2$, and $\Theta_p = 1$ ($s_p = 0.25$) was used to verify the existence of a self-similar solution. The problem of the emergence into a self-similar mode of a system of equations with non-self-similar initial data was solved numerically for this case. The result for the temperature $T(r, t)$ is presented in Fig. 1. The emergence of the solution into an analytical solution (denoted by an arrow) as $t \to t_f$ is seen. In this example $t_f = 1$.

4. Thermal Structures

The time dependence of the entropy is

$$\exp(S/c_V) \sim \sum = p\rho^{-\gamma} = M_0^{1-\gamma}(v_0 t^n)^{2/n_*} t^{-2}\beta\delta^{-\gamma}.$$

A necessary condition for the appearance of nonmonotony in the temperature profile is the increase in entropy with time and the presence of heat sources (or the decrease in entropy with time and the presence of heat sinks, such as volumetric emission). The requirement of an increase in entropy in the case of compression problems ($-\infty < t < 0$) leads to the condition of compression with large accelerations: $n < n_* \equiv 2/[2 + (N + 1)(\gamma - 1)]$. From this it follows that in self-similar problems of the compression of a fully ionized plasma the appearance of structures is possible only when $\gamma < \gamma^* \equiv 1 + N/2(N + 1)$. Small values of γ can be treated as an effective means of allowing for contamination of the plasma by heavy impurities. Structures connected with the presence of volumetric emission from the plasma, conversely, appear in the problem of compression with $n > n_*$. For problems of dispersion of the plasma in the S-mode ($0 < t < \infty$) the signs of the inequalities in the conditions cited above are reversed.

An analysis of analytical solutions and numerical calculations of self-similar problems shows that an additional condition for the existence of temperature inhomogeneities in the S-mode is the requirement that the characteristic mass depth Δx_T of the thermal skin (it does not depend on time in the S-mode) be less than that of the mass being compressed. The number of temperature maxima is inversely proportional to the quantity $K_T = \Delta x_T/M_0 \simeq \sqrt{|\tilde{\varkappa}|\delta\lambda^{2N}}$, where $\tilde{\varkappa}$ is the dimensionless coefficient of thermal conductivity. In the case of $n = 4/(4 + N)$ we have $\tilde{\varkappa} = A_1\Theta^{5/2}$, $A_1 = \alpha_1 v_0^{4+N}/M_0 R^{7/2}$. An analysis of numerical calculations gives a more accurate estimate of the skin depth:

$$\Delta\lambda \simeq 5\sqrt{\frac{\tilde{\varkappa}(\gamma-1)}{2(n/n_* - 1)\delta}}.$$

The expression for the coefficient of thermal conductivity is

$$\varkappa = aT^m\rho^k = \frac{M_0 R\tilde{\varkappa}}{(v_0 t^n)^{N-1} t} = \frac{M_0 R}{(v_0 t^n)^{N-1} t} A\Theta^m\delta^k,$$

$A = \alpha v_0^{2m+N-1-k(N+1)} M_0^{k-1} R^{-(m+1)}$. If the dimensionless temperature Θ and density δ in the expression for $\tilde{\varkappa}$ are averaged over the interval $\lambda_1 < \lambda < \lambda_2$, then the number of temperature maxima in this interval is

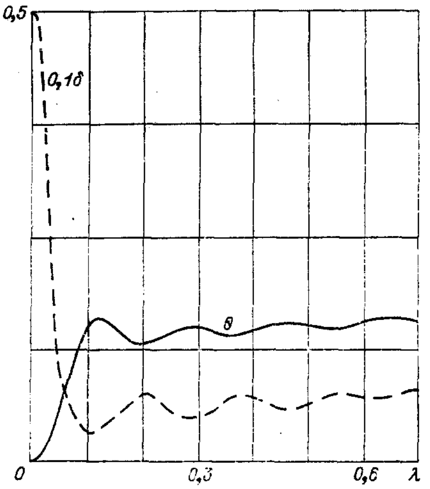

Fig. 3

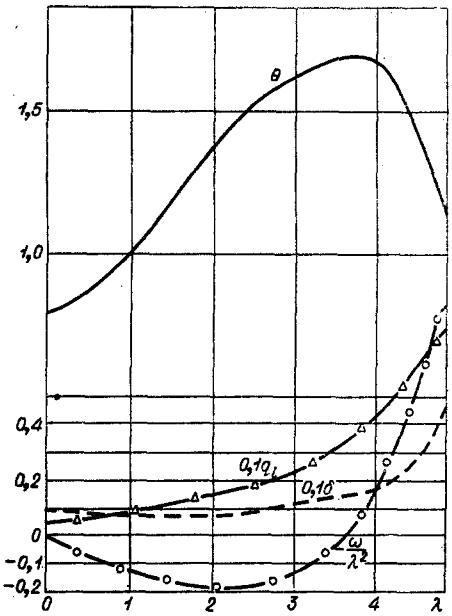

Fig. 4

$$N_T \approx \frac{\lambda_2-\lambda_1}{\Delta\lambda} = \frac{(\lambda_2-\lambda_1)\sqrt{2(n/n_*-1)\,\delta}}{5\sqrt{\varkappa(\gamma-1)}}.$$

5. Some Numerical Solutions of the System of Dimensionless Equations

For a fully ionized plasma a self-similar solution with separable variables is possible when $n = 4/(4+N)$. In the case of $N = 2$ we obtain $n = 2/3$. In this case the time dependences of the quantities have the following form: $v \sim t^{-1/3}$, $p \sim t^{-2}$, $T_{i,e} \sim t^{-2/3}$, $W_{i,e} \sim t^{-5/3}$, and $r_p \sim t^{2/3}$, where $-\infty < t < 0$. The results of numerical integration for different values of γ and of the dimensionless dissipation coefficients are given in Figs. 2-7, where the dimensionless quantities are shown as functions of the dimensionless Eulerian coordinate $\lambda = r/v_0 t^n$. Figure 2 illustrates the compression of a one-temperature viscous and heat-conducting medium for the case of $\gamma = 1.2$. In this case $n_* = (10/13) > n$ and the compression takes place with an increase in entropy. The coefficient of thermal conductivity is $\varkappa = 10 \cdot v_0^{-6} M_0 R^{7/2} T^{5/2}$ and the coefficient of second viscosity is $\zeta = 5 v_0^{-6} M_0 R^{5/2} T^{5/2}$. For the given calculation the dimensionless mass being compressed is $s_p = \int_0^{\lambda_p} \delta\lambda^N d\lambda = 0.17$. The maxima of the temperature Θ are formed because of the superheating instability due to viscous dissipation of energy. The source of the heat dissipated by viscosity ("viscous" heating) is maximal in regions of a maximum in Θ. The density δ is minimal in these regions. The result of the numerical solution of the system of self-similar equations for the case of compression of a one-temperature heat-conducting medium with allowance for volumetric emission is presented in Fig. 3. Here $\gamma = 5/3$, i.e., $n_* = 1/2 < n$, and the compression takes place in the mode of a decrease in entropy. The coefficient of thermal conductivity is $\varkappa = 0.0001 v_0^{-6} M_0 R^{7/2} T^{5/2}$, while the term modeling the volumetric emission is $Q_E = 1.5 v_0^3 M_0^{-2/3} R^{7/2} T^{1/2} \rho^{5/3}$.

The minima of the temperature Θ are caused here by volumetric emission: The heat sink due to emission from the system is maximal in these regions. The pressure β increases monotonically with λ, as follows from the momentum equation. Therefore the density δ is maximal in regions of a minimum of Θ.

Figure 4 illustrates the possibility of the formation of a temperature maximum due to volumetric energy release. In the calculation $\gamma = 1.2$, the coefficient of thermal conductivity is $\varkappa = 0.274 v_0^{-6} M_0 R^{7/2} T^{5/2}$, and viscosity is not taken into account. The term $Q_i = \alpha T_i^{1.6} \rho^{1.3}$ satisfies the condition $n = 2/3$ for the self-similar compression of a fully ionized plasma. In the calculation presented $Q_i = v_0^{-0.3} M_0^{-0.3} R^{1.6} T_i^{1.6} \rho^{1.3}$. In the given example the maximum of the temperature Θ is caused by a superheating instability due to the source $q_i = \Theta^{1.6} \delta^{1.3}$. The possibility in principle of the formation of a superheating

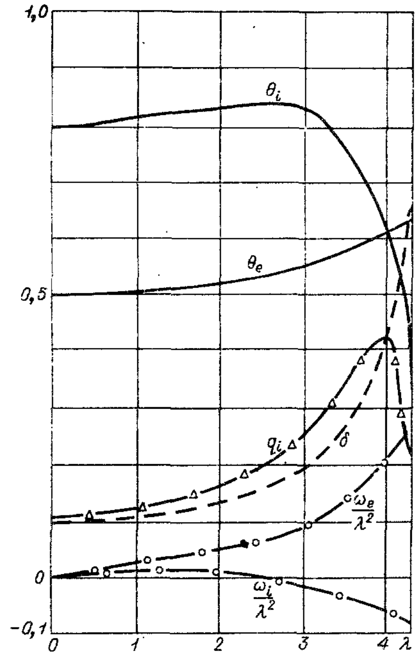

Fig. 5

instability in a medium with viscosity or a volumetric source was pointed out in [15, 16]. The examples illustrated by Figs. 2 and 4 confirm this possibility.

The results of a numerical solution of the problem of the compression of a two-temperature plasma with allowance for volumetric energy release are presented in Fig. 5. Here $\gamma = 1.2$, the coefficients of thermal conductivity are $\varkappa_e = 10 v_0^{-6} M_0 R^{7/2} T_e^{5/2}$ and $\varkappa_i = 0.274 \cdot v_0^{-6} M_0 R^{7/2} T_i^{5/2}$, the coefficient to the volumetric term is $\xi = 0.944 v_0^6 M_0^{-1} R^{-1/2} T^{-3/2} \rho^2$, the term modeling the volumetric energy release is $Q_i = 3 v_0^{-0.3} M_0^{-0.3} R^{1.6} T_i^{1.6} \rho^{1.3}$, and the multiplicity of ionization is $z = 1$. In the solution the maximum of energy release does not lie at the center. This mode models the compression of the central section of the plasma (from the center to the front of the thermal wave). The heat from the thermal wave travels into the central region by electronic heat conduction. The density peak at the piston models the density peak traveling ahead of the thermal wave. Separation of the temperature is observed in the central region.

6. The S-Mode in the Presence of a Magnetic Field (Separation of Mass and Time Variables in the Magnetohydrodynamic Equations)

When a magnetic field is present in the plasma the equation of diffusion of the magnetic field is added to the system of equations (sec. 1), the Lorentzian force is introduced into the momentum equation, and Joule heating is introduced into the energy equation. The coefficient of electronic thermal conductivity begins to depend on the magnetic field:

$$\varkappa_e = a_1 T_e^{m_1} \rho^{k_1} \left(1 + a_{11} T_e^{m_{11}} \rho^{k_{11}} \left(\sqrt{H_\varphi^2 + H_z^2}\right)^{b_1}\right)^{-1}.$$

Here the cases of N = 0 and 1 are analyzed, with the problem being considered as cylindrically symmetrical in the latter case. The coefficient of magnetic viscosity is

$$\nu_m = a_{12} T_e^{m_{12}} \rho^{k_{12}} \left(1 + a_{13} T_e^{m_{13}} \rho^{k_{13}} \left(\sqrt{H_\varphi^2 + H_z^2}\right)^{b_2}\right)^{-1}.$$

The quantities m_i and k_i introduced satisfy the condition of self-similarity:

$$n = \frac{2m_{11} + b_1}{2m_{11} - k_{11}(N+1) + (1-N)\frac{b_1}{2}} = \frac{2m_{12} - 1}{2m_{12} - k_{12}(N+1) - 2} = \frac{2m_{13} + b_2}{2m_{13} - k_{13}(N+1) + (1-N)\frac{b_2}{2}}.$$

The time dependence of the magnetic fields is

$$H_{z,\varphi}(x, t) = M_0^{1/2} (v_0 t^n)^{\frac{1-N}{2}} t^{-1} h_{z,\varphi}(s)$$

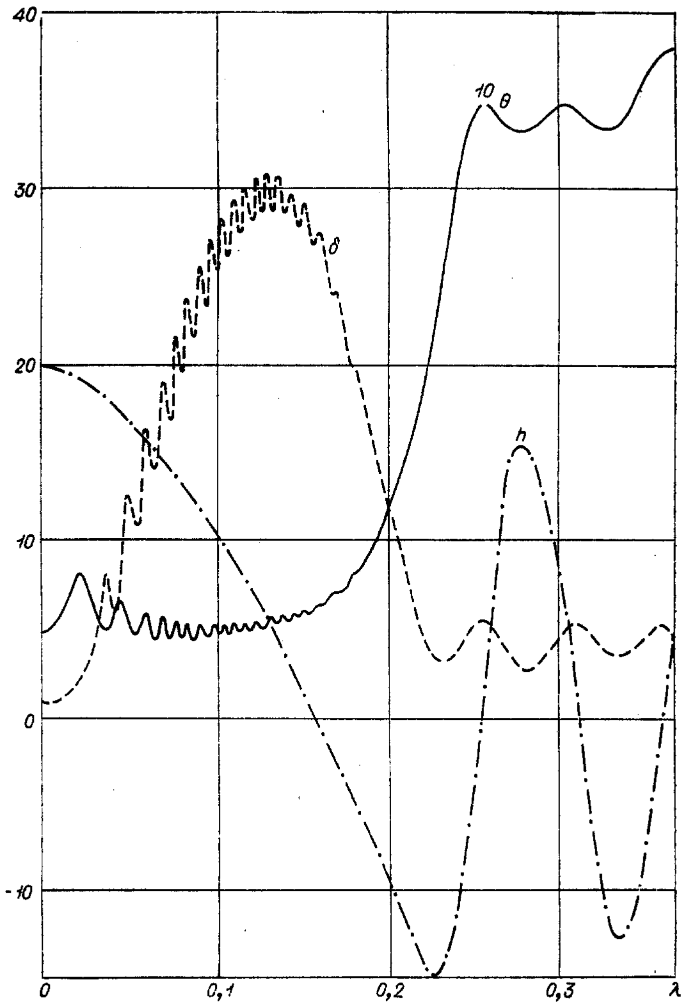

Fig. 6

and the total current in the Z-pinch is

$$I = I_0 t^{n-1+(1-N)n/2}.$$

The conditions for the formation of "magnetic structures" are analogous to those cited in Sec. 4. Thus, the condition of a decrease in the magnetic flux $F_z \sim \int H_z r^N dr$ with time will be a necessary condition for the nonmonotony of the axial magnetic field (h_z). For the problem of compression ($-\infty < t < 0$) this requirement leads to the condition $n > n_\Theta \equiv 2/(3 + N)$, which is always satisfied in the case of the S-mode for a fully ionized plasma [$n = 4/(4 + N)$]. In particular cases one is able to construct an analytical solution with an oscillating field.

The characteristic mass depth Δx_M of the magnetic skin is determined similarly to Δx_T. Cases when $\Delta x_M \lessgtr \Delta x_T$ are possible. The number of maxima of the magnetic field is inversely proportional to the quantity

$$K_M = \Delta x_M/M_0 \simeq \sqrt{|\tilde{\nu}_m|\delta^2\lambda^{2N}}.$$

Profiles of the dimensionless temperature $\Theta = RTt^2/(v_0 t^{4/5})^2$, the dimensionless density $\delta = \rho(v_0 t^{4/5})^2/M_0$, and the dimensionless axial magnetic field $h_z = H_z t/\sqrt{M_0}$ along the dimensionless spatial coordinate $\lambda = r/(v_0 t^{4/5})$ in the problem of the compression by a piston (the coordinate of the piston is $\lambda_p \cong 0.4$) of a finite mass of plasma (a Θ-pinch with a laser) are shown in Fig. 6. The electronic thermal conductivity $\tilde{\varkappa} = -0.0001\Theta^{5/2}$ and the finite conductivity $\tilde{\nu}_m = -0.001\Theta^{-3/2}$ in the plasma are taken into account, $\gamma = 1.2$, $N = 1$, $n = 4/5 \cdot (n < n_*)$, and the dimensionless dissipation coefficient are negative, since $-\infty < t < 0$. The dimensional coefficients of thermal conductivity and conductivity are positive and $\lambda = 0$ is the axis of symmetry. A strong density peak, located near the first minimum of the magnetic field (the ρ-layer) is observed in the region $\lambda \in (0.02-0.18)$. It is mottled by a series of

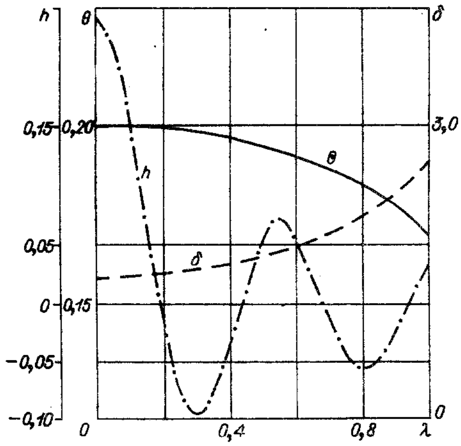

Fig. 7

temperature maxima and density minima — a series of current layers. The magnetic field differentials in the current layers are in the third decimal place and are not noticeable in the graph. Taking real values of $\varkappa_e$ and ν_m for a fully ionized hydrogen plasma we find, using the equations of a dimensional analysis (see Sec. 5, for example), that for the given calculation the mass compressed is $M_p \simeq 0.074$ g/cm. At $t = -10^{-4}$ sec the radius of the pinch is $r_p \simeq 70$ cm and the average density is $n \simeq 10^{18}$ cm^{-3}. At $t = -10^{-5}$ sec, $r_p \simeq 10$ cm, $n \simeq 10^{20}$ cm^{-3}, and the piston velocity is $v_p \simeq 10$ km/sec. The magnetic field at the maximum is $H_{max} \simeq 6.7/t$ Oe; at the time $t = -10^{-4}$ sec, $H_{max} \simeq 70$ kOe.

Let us present one more example of nonmonotonic behavior of the magnetic field. Profiles in dimensionless form of the axial magnetic field $h_z = H_z t/\sqrt{M_0}$, the plasma temperature $\Theta = Rt^2T/(v_0 t^{4/5})^2$, and the density $\delta = [(v_0 t^{4/5})^2/M_0]\rho$ along the dimensionless spatial coordinate $\lambda = r/v_0 t^{4/5}$ are presented in Fig. 7. The axis of symmetry corresponds to the value $\lambda = 0$; $\lambda = 1$ corresponds to the piston compressing the plasma. The electronic thermal conductivity and the finite conductivity are taken into account: $\tilde{\varkappa} = -185.5\Theta^{5/2}$, $\tilde{\nu}_m = -0.00032\cdot\Theta^{-3/2}$. Since the dimensionless temperature Θ varies slowly along λ, one can obtain an analytical solution for $h(\lambda)$: $h = h(0)J_0(\lambda\sqrt{(1-2n)/\tilde{\nu}_m})$, where J_0 is a Bessel function and $n = 4/5$ (for a fully ionized plasma). The ideal equation of state with $\gamma = 5/3$ was used:

$$\widetilde{\nu}_m = \nu_m t/(v_0 t^{4/5})^2 \simeq \text{const.}$$

From the form of the solution it follows that a nonmonotonic magnetic field profile can develop in the case of a large enough plasma conductivity ($\tilde{\nu}_m$ is small) and with the condition that the compression of the plasma takes place without very large acceleration or deceleration ($n > 1/2$). These two indications coincide with the conditions for the formation of reverse currents in pinch experiments. The real parameters of the given calculation are the following: $M_p \simeq 2\cdot10^{-7}$ g/cm, at the time $t = -10^{-4}$ sec $r_p \simeq 250$ cm and $v_p \simeq 20$ km/sec; at the time $t = -10^{-5}$ sec, $r_p \simeq 40$ cm, $v_p \simeq 30$ km/sec, and $n \simeq 10^{14}$ cm^{-3}. The field at the center at $t = -10^{-4}$ sec is $H_{max} \simeq 1$ Oe.

In summing up the research in [7-13] let us dwell on some unique properties of S-modes of compression of a finite mass of plasma.

With fixed properties of the medium the S-modes represent the boundary between faster modes of increase in the quantities at the boundary, resulting in the ordinary increase with time in the depth of penetration into the plasma of thermal, magnetic, and shock waves ("increasing skin") and the slower boundary modes in which the effective penetration of the waves into the medium contracts ("contracting skin"). In the S-mode the mass skin depth does not depend on time [40]. In an established S-mode the medium reacts to the effect of the boundary mode as a single unit.

Unique conditions can be attained in S-modes with real physical parameters of the medium: spatial constancy of the pressure when the velocity of sound is finite (the case of $n = 1$); spatial constancy of the temperature when the coefficient of thermal conductivity is finite (in [7] the case of compression of a heat-conducting medium with the condition $n = n_*$ of constancy of entropy); heating of the medium which does not produce its hydrodynamic motion (the case of $n = 0$ in [7]).

In S-modes the principle of local action occurs (no wave of finite amplitude), owing to which the S-modes possess a distinctive inertia, despite the presence of a number of dissipative processes in the medium. This makes it possible to keep the S-modes active through inertia in the central parts of the plasma for a certain time even after the departure of the boundary modes which produced them.

7. Modes with Peaking for the Equation of Heat Conduction with a Source

It is interesting to find out what results from the effect on the plasma of boundary modes with peaking, when the exponent n does not coincide with that obtained from the conditions of self-similarity for the S-mode. In order to discover the most important aspects of the phenomenon we will discard the complicated system of gasdynamic equations and consider one quasilinear equation of heat conduction in which the coefficient of thermal conductivity is a power-law function of the temperature.

The phenomenon of the metastable localization of heat in a medium with nonlinear heat conduction was studied in [10-13, 40, 41]. The study was conducted on the basis of an analysis of the simplest problems for equations of nonlinear heat conduction in a stationary medium. One of the directions of the investigations consisted in the study of the penetration of thermal waves into a cold half-space (the coefficient of thermal conductivity is reduced to zero in the background). This process was studied earlier in [42-48] for boundary modes not displaying the property of peaking. In these reports the conditions for the existence of a finite front of the thermal wave were determined and a number of properties of nonlinear heat conduction in a compressible medium were studied. In [10-13, 40, 41] the temperature at the boundary increased in a mode with peaking, which models the action of laser radiation on the medium.

An S-mode was constructed (separation of the independent variables) leading to the penetration of the thermal wave to a finite depth and then its stopping. Although energy continues to travel into the medium and the temperature in the zone of localization grows without limit, propagation of heat into the cold medium does not occur. The reason is the "concave" nature of the temperature profile produced by the indicated boundary mode (a more exact definition of when the velocity of the thermal wave is reduced to zero and when it is different from zero is given in [14]).

A self-similar solution in which the half-width of the region of heating contracted with time (LS-mode) was constructed in the case when the temperature at the boundary increased in a mode with peaking but it increased slower (as $t \to t_f$) than in the S-mode.

Profiles of the temperature T over space r at different times are presented in Fig. 8. The half-width, which contracts with time after the mode is established, is marked with crosses.

The successive equal portions of heat, entering in ever shorter time intervals, are localized near the heating boundary.

A number of interesting theorems were formulated and proven in [40]. It was shown that localization of the heat is characteristic of a certain class of boundary modes (S- and LS-modes) which operate with peaking. It was possible to prove the metastable localization of heat for the Cauchy problem and to extend these results to multidimensional problems. Estimates which were made show that the localization of heat promotes the obtainment of superhigh temperatures with comparatively small laser radiation energies: $T \sim 10$ keV in a medium of D + T with $n = 10^{20}$ cm^{-3} with an energy of several hundred joules when the total time of the laser pulse is 10^{-9} sec and the depth of localization is $r_l = 0.1$ cm. It is required that the laser flux, which increases in the peaking mode, reach a limiting value of $q_M \cong 10^{16}$ W/cm^2 at the final time. The confinement time at this temperature comprises several tens of picoseconds [$(20\text{-}30)\cdot 10^{-12}$ sec]. The Lawson criterion is not satisfied in this case, since mainly electrons are heated and the confinement times are small. A number of other physical factors (hydrodynamics, burnout, etc.) can be neglected in accordance with the analogous estimates of [1].

It is important that one can obtain supertemperatures (a hundred million degrees) with meager energy expenditures.

The structures which develop in a plasma when modes with peaking act at its boundaries owe their origin to the presence of volumetric sources or sinks in the medium (Secs. 4-7).

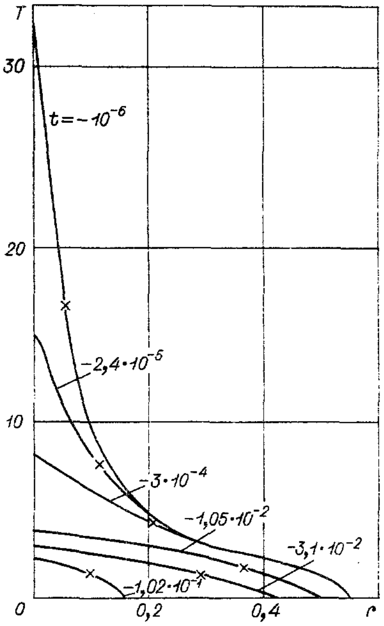

Fig. 8

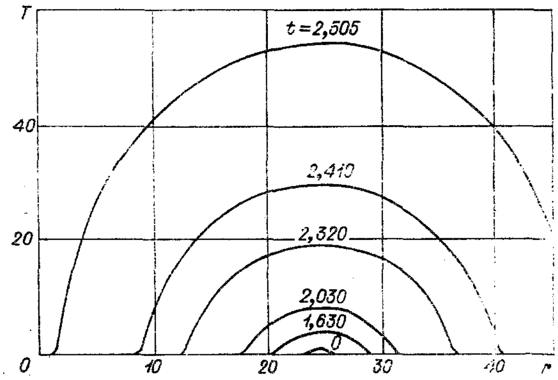

Fig. 9

In addition, under certain conditions nonlinear volumetric heat sources can by themselves lead to the formation of modes with peaking in the medium and, consequently, to the localization of heat and the formation of structures. Such effects have been observed, for example, in work on the study of the current-layer effect [21, 22, 26, 31]. The Cauchy problem for a quasilinear equation of heat conduction with nonlinear volumetric heat sources was studied in [49] for an understanding of this phenomenon and the determination of its properties.

Three modes of burning of a medium described by such an equation are presented in Figs. 9-11. The nature of the temperature dependence of the coefficient of thermal conductivity $\varkappa \sim T^m$ and of the volumetric energy release $Q_i \sim T^l$ determines a mode with an increasing half-width of the burning region for $l < m + 1$ (HS-mode) (Fig. 9), with a constant half-width for $l = m + 1$ (S-mode) (Fig. 10), and with a contracting half-width for $l > m + 1$ (LS-mode) (Fig. 11). The time shift $t \to t - t_f$ described above was used everywhere in the solutions and their numerical illustrations analyzed here and below. The initial data were assigned at the time $t = 0$ while the moment the temperature goes to infinity (the moment of focusing) corresponds to the time $t = t_f$. In all three cases the burning was induced by the introduction of a finite temperature disturbance into a section of initially cold medium. Burning localized by nonlinear processes to a certain limited region of the medium develops in the cases of S- and LS-modes. The nonlinear sources cause the formation of a certain "concave" temperature profile on a characteristic spatial scale called the fundamental length $L_T \sim \sqrt{\varkappa T/Q_i}$ [49]. The region of burning induced by the initial temperature disturbance introduced into the cold medium spreads out in the beginning to the size of the fundamental length. At this size a sharp acceleration of the process takes place (by four to five orders of magnitude in time) and a distinctive burst of energy release develops (the analog of a chain reaction, but only for the case of a nonlinear medium). The growing energy release forms a "concave" temperature profile which does not propagate beyond the limits of the fundamental length for a certain finite time [41, 49]. The developed asymptotic stage of these events is described with the help of self-similar solutions for modes with peaking. A periodic analytical solution was constructed in [49] showing that burning in the S-mode can lead to the breakup of the medium into periodic thermal structures which grow in accordance with a definite law. This solution indicates the formation of a distinctive thermal self-insulation of the separate sections of the medium from each other in the process of burning with peaking.

The result of a calculation showing the independent burning of thermal structures in neighboring fundamental lengths is presented in Fig. 12. Allowance for a number of physical factors (burnout, variation of reaction rate with temperature, etc.) does not destroy the localization phenomenon itself but only limits the time of growth of the thermal structure, accompanied by contraction of its half-width. The burning stage with the spreading of heat

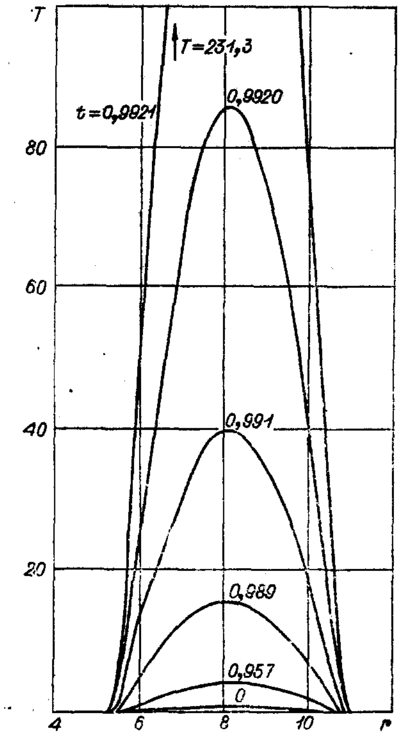

Fig. 10

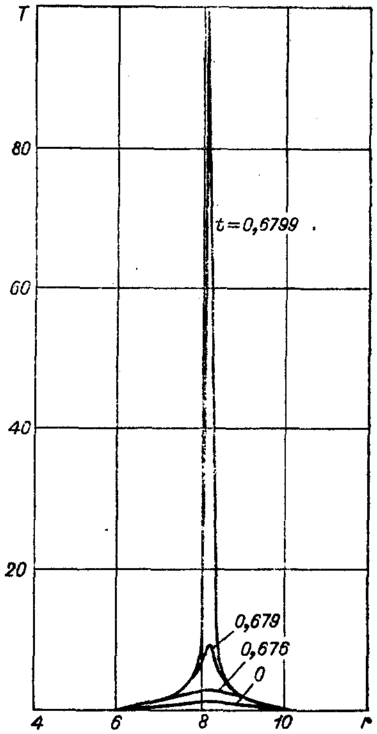

Fig. 11

sets in later. The phenomenon of the localization of heat in a medium with nonlinear heat conduction has been connected earlier with the action of volumetric heat sinks in the medium [50, 51]. The sinks develop, for example, through cooling of the medium during its expansion or through volumetric emission from the plasma. In a medium with sinks structures can also exist under steady conditions [52, 53].

The combustion of a medium uniformly heated at the initial time was also analyzed in [41, 49]. It was shown that it is unstable against small disturbances for the S- and LS-modes. The developed nonlinear stage of this instability in the case of the S-mode is described by the same self-similar solution as for a finite disturbance of amplitude.

The mechanical analogy of this mode with the dynamics of a point in a potential force field makes it possible to find the spectrum of lengths of structures developing in the medium. It is shown that in the developed asymptotic stage this spectrum always degenerates into one fundamental length, i.e., the burning of the medium in a structure for $l = m + 1$ (S-mode) always takes place at a fundamental length L_T. This is illustrated by Fig. 13. The width of the temperature peak which develops corresponds to the fundamental length of the S-mode. It is possible that the different manifestations of the effect of localization of heat (thermal inertia) open up entirely new approaches to the obtainment of CTS (controlled thermonuclear synthesis). In fact, the main problem in CTS — the obtainment and confinement of superhigh temperatures — turns out to be deeply connected with the creation of modes with peaking in the medium, the formation of thermal structures, properties of the quasi-linear transfer equations, and the effect of nonlinear sources.

It is noteworthy that nonlinear processes in a continuous medium thermselves create modes with peaking. In this case it is not necessary that the rise of the boundary modes assigned from without take place with peaking. The study of the current-layer effect [20-33] and of the problem of burning in a medium with nonlinear volumetric sources [41, 49] and calculations and self-similar solutions for systems of nonlinear equations [7, 9] show that modes with peaking can be created in a medium owing to nonlinear relationships in the systems of equations. In fact, the thermal self-insulation of the structures in LS- and S-modes leads to the independence of the burning of the structures in the medium from the action of the boundary mode (if it varies slower with time than the mode produced by the sources).

The problems discussed show that deep internal relationships exist between the nonlinear processes in a medium, its breakup into separate structures, and the distinctive thermody-

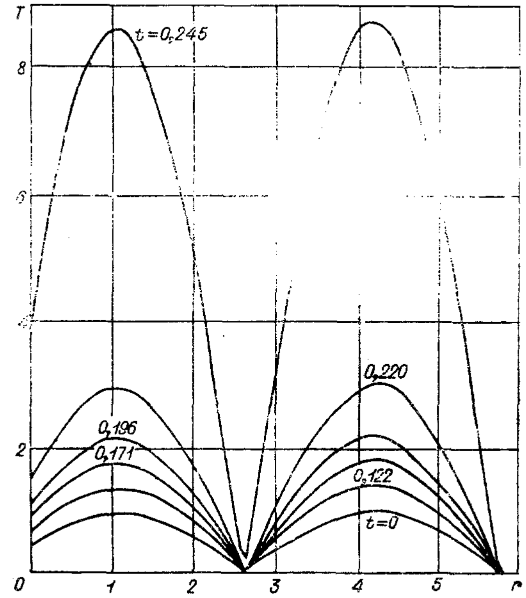

Fig. 12

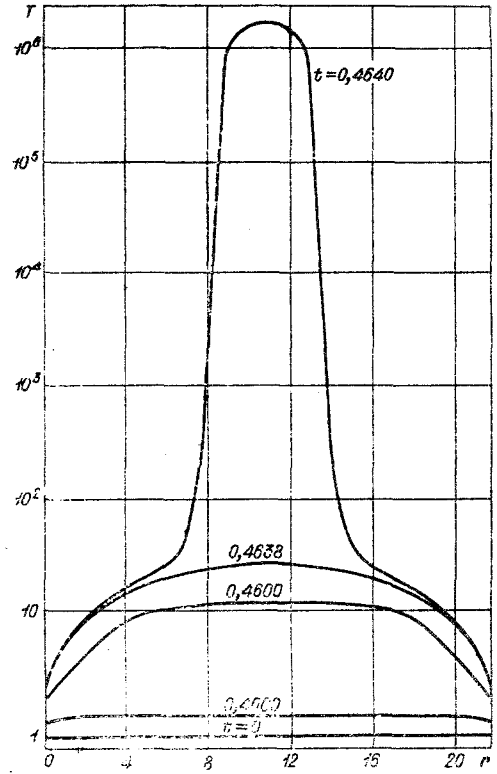

Fig. 13

namics of the modes of peaking which are accompanied by complication of the organization of the medium and the appearance of singular physics of a plasma with structures [7, 9-11, 27]. The transfer processes in such a medium, the conditions of initiation of the thermonuclear reaction, the stability, and a number of other properties are altered cardinally. The prospect arises of using fine nonlinear effects to obtain new approaches to the solution of the CTS problem. These phenomena also have great theoretical importance apart from the possibility of their use in the CTS problem.

The results of the analysis of modes with peaking made it possible to formulate several new principles characterizing the properties of strongly nonsteady processes [41].

1. In the presence of a certain temperature dependence of the coefficients of thermal conductivity of the medium nonlinear heat sources provide for variation of the quantities in the medium in a peaking mode. The metastable localization of heat at definite spatial scales, called the fundamental length [49], occurs for S- and LS-modes of peaking. This provides for the development of thermal structures in the medium at the fundamental lengths. In a compressible medium the fundamental mass plays the role of the fundamental length [11, 12].

2. Structures can coexist if they have the same time of focusing. It is determined by the form of the initial disturbance, particularly by its maximum amplitude. Different initial data leading to the formation of structures with correspondingly different focusing times (and different initial amplitudes T_{mi}) can coexist as a single formation, now with a new focusing time, if there exists a self-similar solution combining this entire set of initial data. Such a combination is allowed only for a certain discrete set of fundamental masses and amplitudes T_{mi} of the structures. Thus, a process of complication of the organization of the medium (the combining of structures into a new single organization takes place) and the principle of superposition of nonlinear systems can be accomplished.

3. Allowance for a number of processes acting simultaneously in the plasma leads to an increase in the number of types of structures and to an enormous variety of self-similar modes with nonmonotonic spatial distributions of the quantities, making it possible to establish the conditions for the combining of the most varied structures (see [7, 9-11, 27, 54], for example). Consequently, in the strongly nonsteady thermodynamics of modes with peaking there appears the inherent possibility of formulating the concept of the existence of a discrete set of fundamental masses of structures which can be combined into a new organization.

It was shown in [7-13] that the interaction of the structures entering into the self-similar solutions of the problem of the compression of a finite mass of medium (S-mode) is accomplished through small sonic disturbances.

4. In LS-modes structures moving together with waves can appear within the fundamental length and the contraction of the half-width of the structures with time occurs [41, 49]. For LS-modes there is an analog of the uncertainty principle connecting the region of localization of the structures with the magnitude of the external action [41, 49]. In the S-mode the localization depends only on the properties of the medium and the structures develop while retaining a constant half-width.

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ADIABATIC COMPRESSION OF A GAS BY MEANS OF A SPHERICAL DRIVER

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§1. A spherical driver* with an initial radius r_0 within which there is a gas at rest ($\varkappa$ is the polytropic index; c_0 is the velocity of sound) starts to converge toward the center at a certain time. The problem is the determination of that driver trajectory for which all β characteristic curves emerging from it converge at the center of the time of collapse of the driver, which is taken to be the origin of the time scale, t = 0, in the following. In this case the motion of the gas within the driver will be spherically symmetric, isentropic, and self-similar. We take $\eta = c_0 t/r$ as the self-similar variable, and the gasdynamical functions are represented in the form

$$u = r/tu_1(\eta); \quad c = r/tc_1(\eta).$$

In the r–t plane, the flow will be separated from the region at rest by the characteristic curve $r = -c_0 t$ ($\eta = -1$). The functions $u_1(\eta)$ and $c_1(\eta)$ are defined by the equation

$$\frac{du_1}{dc_1^2} = \frac{u_1\left[(u_1-1)^2-3c_1^2\right]}{2c_1^2\left[(u_1-1)(\varkappa u_1-1)-c_1^2\right]} \qquad (1.1)$$

*A solution is given in [1] for the case of a plane driver. A self-similar spherically symmetric compression wave was also considered by I. E. Zababakhin and V. A. Simonenko. (Private communication — Ya. K.).

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